Measuring the mustard seed: an exercise in indirect measurement and mathematical modelling

Amit Dhakulkar, Nagarjuna Gadiraju and Samir Dhurde

ABSTRACT As a first exercise for middle or high school students in indirect measurement using physical and mathematical modelling, we present here a simple task where students are asked to find the average diameter of mustard seeds. The resulting observations lead to a simple linear mathematical model which has an accessible physical basis in the real world. This simple task also provides a rich opportunity and a context for learning several topics in measurement, modelling, use of graphs and statistics. We present this as a template to be used for developing a series of activities for learning indirect measurements and physical and mathematical models.

Once students have learned about measuring dimensions in real life using direct methods, it is important that they learn how to measure very small and very large dimensions or dimensions that are not directly accessible, such as large trees or the distance between the Earth and the Sun, or very small objects such as microbes, pollen or the thickness of a hair. Under these situations, scientists use *indirect measurement techniques*. These often require the use of alternative physical models and corresponding mathematical models. For example, to measure the height of a building, the alternative physical model may be an analogous triangle and the mathematical model would be the equations describing the proportionality. In some cases, even the physical model may not be amenable to direct experience or direct measurement. In such cases, mathematical models provide a way to measure the required dimensions that often depends on assumptions and approximations about the physical world,

In order to prepare students to understand this core aspect of scientific literacy, we have designed learning contexts that help students understand physical and mathematical models. This article describes one such preliminary exercise in physical and mathematical modelling, in which both types of model are directly amenable to experience and measurement. The task is designed to stimulate discussions on a variety of concepts as shown in the mind map in Figure 1.

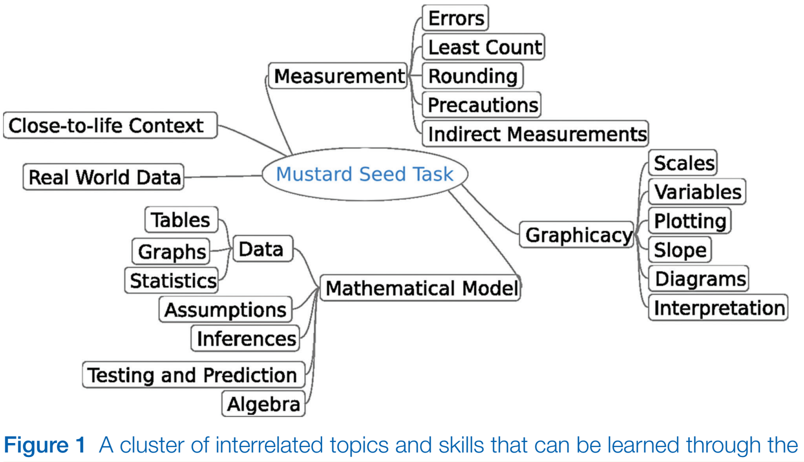

Figure 1 A cluster of interrelated topics and skills that can be learned through the mustard seed task

Mustard seeds

The Indian kitchen is a versatile place where many spices and ingredients mingle to produce a variety of cuisines. Although each part of India has a unique style of cooking, there are many things that you will find in all kitchens. One of them is the mustard seed, of the genus *Brassica*. In many of the cuisines, the mustard seed is essential to give a *tadka* or flavour to the food. In many Indian languages, the mustard seed is metaphorically used to denote something of a very small size. There are three species that are commonly found and all of them are so small that they are difficult to measure with a scale. Two of them (*Brassica juncea* and *B. alba*) have seeds almost double size of the other (*B. nigra*), as shown in Figure 2. Variation in the size and shape in the seeds of the same species can also be seen in this figure. These two characteristics of the seeds provide opportunities for classroom discussion, as we will see later.

The designed task was field tested in three consecutive summer camps for class VIII students (age 13–14 years) from a variety of urban Indian schools. The task was performed over two days. On the first day, a few different possible approaches to measure the diameter of the mustard seeds were discussed with the students. The students were to do actual measurements of seeds in their homes. Also, they were asked to consider how and why a mathematical model (both algebraic and graphical) may fit the observations. Students were asked to write detailed reports on the task.

On the next day, data were collected from all the students and plotted on a projector. An interesting pattern emerged from this collaborative plot. The mathematical model was again discussed along with the data from all the students, which helped in understanding the physical meaning of terms in the mathematical model. In what follows, we narrate the significant events when interesting learning took place to highlight how a simple task can be so rich in its learning outcomes.

Figure 2 Mustard seeds commonly used in India, and their sizes; from left to right: *Brassica juncea*, *B. alba* and *B. nigra*

Warm-up discussions

All the students were familiar with mustard seeds. Two examples that are similar to the mustard seed task involving indirect measurement were discussed in the camp. One of the tasks was the indirect measurement of the width of a thread. This is usually done by winding the thread on an object (for example a pencil) and finding the width for a given number of turns. We would get the average width of the thread by dividing this length by the number of turns. The second task discussed was how to find the thickness of one page of a book. This task also involves measuring the thickness for a given number of pages and then the average thickness is found by dividing the total thickness by the number of pages.

After either of these two warm-up discussions, the students were asked to guess the approximate diameter of one mustard seed. For this purpose, some mustard seeds (*B. juncea*) were shown to them. During the discussions that ensued, the students came up with guesses from a few millimetres to a few centimetres.

1. The word “either” suggests that any particular student would only attend one discussion or the other discussion, but not both. Is that correct? If so, I suggest rephrasing this as something like “After participating in one or the other of these two warm-up discussions,” – OK?

How to measure the mustard seeds

In the next part of the discussion, the students were encouraged to come up with ideas for measuring the diameter of a seed. Some of the students devised ingenious methods of measuring the diameter. One of the students suggested that a thread should be wound on the seed, and then the length of the thread could be measured easily with a ruler. Another student suggested an even more elaborate method: we could find out the volume of displacement of water due to one seed and then from that the volume of the mustard seed; from this volume we could calculate the radius and hence the diameter. The students were already familiar with the properties of a circle (area and circumference) and a sphere (volume), which would be used in the above two methods. One of the students said that a divider from a geometry box could be used to hold the seed, and then the distance between the points of the divider could be measured on the scale. Yet another student rolled a seed in a piece of paper, and measured the diameter of the roll. Despite the discussions in the warm-up on similar tasks, it was interesting to note that it never occurred to any student to use more than one seed in the measurement. This may indicate that the transfer of mathematical knowledge from one context to another is often not easy.

The teacher asked the students to look at the mustard seeds and to tell whether all of them were of exactly the same size and shape. As we can see in Figure 2, there is variation in size and shape even in the same species and this was noted by the students. They said that the seeds were not of the same size – some were larger and some were smaller. Some of the students responded to this trigger of variation in size by saying that an average of many values needs to be taken. Thus the rationale of doing multiple measurements on different seeds and taking averages of the readings was brought in. This discussion helped the students to realise almost on their own that variation calls for measuring the averages.

After this, they finally decided to use a scale (on which the smallest scale division was 1 mm) to measure the average diameter. In the discussion that followed, the procedural details of the task were worked out. The method involved aligning a number of mustard seeds along a scale and measuring the length covered by them. Then the average diameter of the seeds in each such set was found. The precautions to be taken were discussed. Figure 3 shows a photograph and a drawing made by one of the students to explain the procedure. The students were guided to take measurements of sets of 5, 10, 15, 20, 25 and 30 mustard seeds, although some students went up to 40 seeds. After this, possible ways of making a mathematical model from these observations was discussed along with the assumptions that were required for such models. The discussion on modelling was further elaborated on the next day, with graphs, and will be discussed later.

Peculiarities in the reports

The students submitted written reports that were supposed to include the procedure, precautions taken, possible errors, observations, tables, pictures and conclusions. The students were also asked to plot a line graph for sets of each number of seeds versus the total length that they measured on the scale.

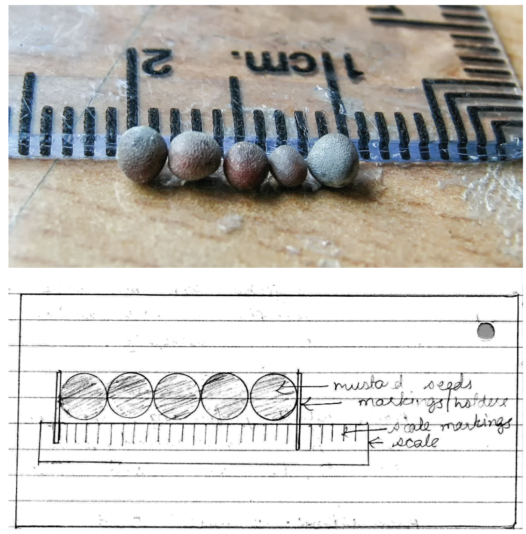

Figure 3 A photograph showing the placement of five seeds along a scale and a drawing by one of the students; placeholders that hold the seeds together can be seen in the drawing

The observations on the different sets of the seeds were recorded in a table that had the following column headings:

- Number of seeds;
- Length in mm/cm;
- Average diameter for 1 seed.

Some students used calculators to get answers up to 4 decimals. This provided an opportunity to discuss significant figures, the concept of least count, and rounding off in the class on the next day. The most commonly reported error was of the alignment and placement of the seeds with the scale. This error was the most irritating for some students although it was fun for others. Although the students did not realise it, the problem was aggravated at times by the presence of static electricity charge on the seeds. One of the students actually glued the seeds on the paper to overcome this problem.

Plotting

Almost all of the students who drew the graph could plot the data sets correctly. Not all of them drew best-fit lines through the points that they had plotted. Some students joined the individual points instead of drawing a single straight line passing through all of them. However, since the data were for a linear function, they still ended up with a more or less straight line. Some of the students drew the graph on plain paper rather than using the graph paper they had been given. Only one student drew both a bar graph and a line graph. The students were not given specific instructions on choosing the scales but they were asked to write the scales on their graphs. While most students chose a scale of 5 seeds per unit for the x-axis, various scales were used for the y-axis. Many students chose the same scale as the actual readings, with 1 mm on the graph paper representing 1 mm of the actual measurements (Figure 4). One of the students plotted the values of the average diameter that was obtained from the measurements against the number of seeds in each set.

Modelling

The next day, the students came with their written reports and they were asked what average diameter they had found. Their answers varied by more than a factor of 2. Through some probing about why the answers varied, it emerged that there were in fact two different varieties of mustard seeds. Students did not realise this initially because each of them had only one of the varieties at their home, and thus had data only for one type. This provided us with an opportunity to discuss what the existence of two sizes would mean for the mathematical model.

During the discussion, the students were asked whether there were any mathematical relationships between the number of seeds and the lengths that they had measured for them. It was noted that, as the number of seeds increases, the total measured length also increases. It was thus agreed that the measured length of the sets of seeds L is perhaps directly proportional to the number of seeds n:

$$L \propto n \qquad (1)$$

After agreement that these two quantities are in direct proportion (they would be if all seeds were equal), the discussion was taken further by introducing a proportionality constant d. Hence the mathematical relation between the two quantities L and n can be written as:

$$L = \mathrm{d}n \qquad (2)$$

At this point, the students were reminded of the straight line equation:

$$y = \mathrm{m}x \qquad (3)$$

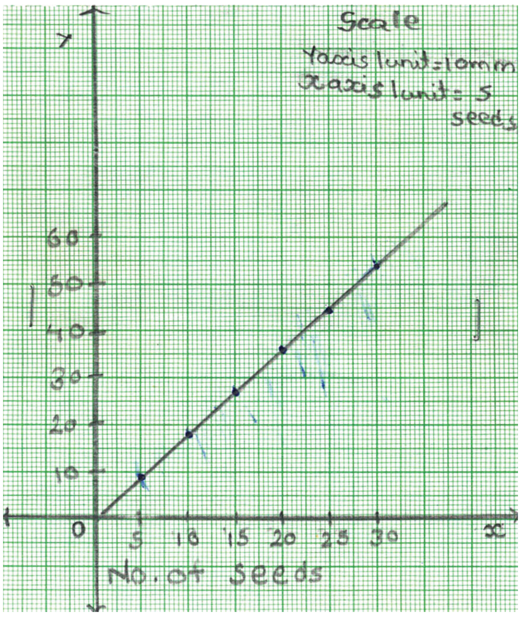

Figure 4 A graph one student made of length vs no. of seeds. Here the y-axis has a scale of 1 mm = 1 mm and the x-axis has a scale of 1 cm = 5 seeds.

where m is the slope of the line. We then compared the two equations for similar terms. The total length L and the number of seeds n in Equation 2 are analogous to the y and the x values respectively in Equation 3. The proportionality constant d in Equation 2 can be seen as analogous to the slope m in Equation 3. All this leads to the fact that Equation 2 is indeed an equation for a straight line, which explains why the students could draw a reasonably straight line passing through all the points.

Collaborative plotting

To underline the understanding of the slope of the line and its meaning in the mathematical model, a collaborative exercise was carried out on the second day. The length of each set of seeds (5 seeds, 10 seeds, 15 seeds, etc.) were collected and displayed on a spreadsheet in *GeoGebra*, a dynamic mathematics software package (http://geogebra.org). Then the average lengths of each of these collected values were plotted against the corresponding number of seeds in each set (Figure 5). When the points were joined, two distinct lines emerged, corresponding to each type of seed. Why did we get two distinct lines?

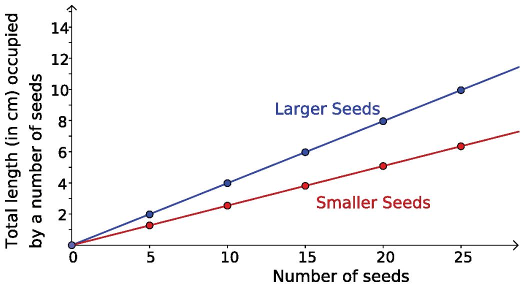

Figure 5 The graphs of lines for two different types of seeds drawn in *GeoGebra* from the average readings by the students; the difference in their slopes is linked to the difference in the size of the seeds

Discussion followed on the physical meaning of the slope of the line. In this case, we have a concrete physical observation that the sizes (diameters) of the two types of seeds are different. In the mathematical model, the slopes of the lines are different. Thus we can relate the abstract change in the slope of the mathematical model to a concrete observation regarding the size of the mustard seed. This point was discussed at length in the camp. It was helpful to use *GeoGebra* to visualise how the lines would have looked if the slopes were different, meaning if the size of seeds were different. For example, if the seeds had an average diameter of 0.5 mm or 3 mm, where would the lines be with respect to the lines drawn? In this way, the use of graphs for understanding the meaning of the slope in terms of associated lengths was made clear. The students were also asked to plot the collaborative average on their own graphs. Drawing this 'average plot' shows how much deviation the students' readings have from the average values. This led to interesting discussions about different aspects of statistics such as averages, standard deviations and need to take multiple measurements.

Predicting

Another use of graphs in the context of modelling is as calculating/predicting devices. The students were shown, by means of an example in *GeoGebra*, how to find the length for a given number of mustard seeds. For example, to find the length for a set of 50 mustard seeds in a row, we need to take a line that is parallel to the length (y) axis and find the point of intersection of this line and the line made from observations. Similarly, we can also find the number of seeds, if we know the length for the set of seeds, by using a line parallel to the number of seeds (x) axis. The students were asked some practice questions for these types of predictions. They solved these questions using both the algebraic and the graphical method, and compared the results with actual observations (for a given number of seeds). The results were in agreement between the methods and the actual observations, within the error margins.

Conclusion

The activity described in this article is part of a larger project on developing critical graphicacy skills. Graphicacy is defined as the ability to understand and present information in the form of sketches, photographs, diagrams, maps, plans, charts, graphs and other non-textual, two-dimensional formats (Aldrich and Sheppard, 2000). In their book *Critical Graphicacy*, Roth, Pozzer-Ardenghi and Han (2005) take the stance that '*our aim as critical educators is not just the provision of opportunities for students to become graphically literate; rather, we want students to develop critical graphicacy, that is, we want them to become literate in constructing and deconstructing inscriptions, the deployment of which is always inherently political*.' In our study on Indian textbooks, we found that the presence of graphs is limited and opportunities to use them are almost non-existent (Dhakulkar and Nagarjuna, 2011). In such a scenario, it is important that opportunities are provided to the students. In this case, the emphasis was on using and understanding graphs in the context of mathematical modelling of 'real world' data and measurements. This can be seen as a first step in the direction of making students graphically critical and literate. This task also provides a concrete context for the students to use multiple symbolic systems (tabular, graphical and algebraic in this case) and to understand the relationships between them.

When designing this activity, we adhered to the following principles:

- the learning context should be close-to-life;
- the constructed model should be abstract enough to find applications in multiple places;
- it should provide opportunities for linking several trans-disciplinary skills;
- it should be easy to do in the classroom;
- and it should not consume too much time.

Alhough the task and the model were simple, not all the students came close to the expected result. Some students could not go on to make the mathematical model on the first day. Only after the discussions on the second day could they do so. This shows that bridging the gap between abstract mathematical knowledge and the real world is not trivial. By making students aware of the fact that the *same* mathematical model can describe different objects, one can perhaps hope to overcome this problem.

It is vital to bring tasks to the classroom tasks that are simple but rich enough to raise discussions of several interrelated concepts in a close-to-life context. Other tasks such as measuring the thickness of paper or the diameter of a thread can be done as a follow-up to this task. This would emphasise the power of mathematical modelling to the students: that using the same general linear model, we can model systems that are not similar to each other. A few such experiments can act as a springboard to scientific modelling, and would help the students find the links between the models and the real world.

Acknowledgements

We acknowledge the inspiring Arvind Paranjpye and the interactions with him about the 'mustard seed puzzle' that he poses to students. Authors are also thankful to Karen Haydock for helpful comments on the article. We also appreciate the help of summer interns at the Inter-University Centre for Astronomy and Astrophysics (IUCAA), for their diligent help in data collection. The study would not have been possible without the facilities provided by IUCAA.

References

Aldrich, F. K. and Sheppard, L. (2000) 'Graphicacy': the fourth 'R'? *Primary Science Review*, **64**, 8–11.

Dhakulkar, A. and Nagarjuna, G. (2011) An analysis of graphs in school textbooks. In *Proceedings of epiSTEME 4 – International Conference to Review Research on Science, Technology and Mathematics Education*, ed. Chunawala, S. and Kharatmal, M. India: Macmillan.

Roth, W. M., Pozzer-Ardenghi, L. and Han, J. Y. (2005) *Critical Graphicacy*. Science and Technology Education Library, vol. 26. Netherlands: Springer.

Amit Dhakulkar is a research scholar at Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research (TIFR), Mumbai, India. Email: amit@hbcse.tifr.res.in

Nagarjuna Gadiraju is a faculty at Homi Bhabha Centre for Science Education, TIFR, Mumbai, India. Email: nagarjun@gnowledge.org

Samir Dhurde is in charge of the Public Outreach Programme at Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India. Email: samir@iucaa.ernet.in

2. In British English, it sounds odd to say that a person is "a faculty", rather than "a faculty member". But if this is normal usage in India, that is fine.